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IDEAS ON A MODIFICATION OF THE SYNCHRONOUS AMPLIFIER TO IMPROVE SENSITIVITY

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IDEAS ON A MODIFICATION OF THE SYNCHRONOUS AMPLIFIER TO IMPROVE SENSITIVITY

1. The Conventional Synchronous Amplifier [1]

As is well known, the direct way to detect a sine-shaped signal of definite frequency and phase in band-limited white noise is by decreasing the bandwidth, because the mean amplitude of the noise voltage is proportional to the square root of the bandwidth. The term 'white noise' means a constant average power density over the frequency range f_1 to f_2 being considered. The term 'band-limited" is taken to mean that power density is zero outside the region f_1 to f_2. That is, an ideal band pass filter with a rectangular pass curve is assumed. Further, let it be assumed that there is so-called narrow-band noise present. That is, let the bandwidth $B = f_2 - f_1$ be small in comparison to the mean frequency $f_0 = (f_2 + f_1)/2$. If this condition is met, then the resulting noise voltage can be observed as a harmonic wave $V(t) = R(t) \cos [2\pi f_o t - \varphi(t)]$. $R(t)$ and $\varphi(t)$ are quantities which fluctuate statistically. $R(t)$ follows a so-called Rayleigh distribution, while $\varphi(t)$ is distributed with constant probability across the complete angular range from 0 to 2π. If the signal likewise has the frequency f_0 and the amplitude c_s, it is necessary to detect $c_s \sin 2\pi f_o t$ in the total voltage

$$V'(t) = V(t) + c_s \sin 2\pi f_o t = R(t) \cos[2\pi f_o t - \varphi(t)] + c_s \sin 2\pi f_o t$$

The ratio of the signal amplitude to the noise amplitude thus becomes more favorable, the narrower the spectral range filtered out around the signal frequency, because, as mentioned above, the amplitude of the noise voltage is proportional to the square root of the bandwidth. But there are limits to the narrowing of the bandwidth because production of very narrow band filters (fractions of 1 Hz) raises difficulties. The principle of the synchronous amplifier is of assistance here. In this operation, the voltage $V'(t)$ is multiplied with a reference voltage $2\sin 2\pi f_o t$. This yields

$$2V'(t)\sin 2\pi f_o t = 2R(t)\cos[2\pi f_o t-\varphi(t)]\sin 2\pi f_o t + 2c_s \sin^2 2\pi f_o t$$
$$= R(t)\sin\varphi(t) + c_s + \text{terms of frequency } 2f_o \quad (2)$$

Thus the alternating signal voltage is transformed into a direct voltage and, in place of the narrow-band filter, a small low-pass filter (i. e., a large measuring time constant) is suitable for averaging out the disturbing noise voltage. If the terms of frequency $2f_0$ are filtered out, the noise voltage is given as $R(t)\sin\varphi(t)$. Measurement with a time constant of τ seconds corresponds (approximately) to removal of a section of the spectrum of $R(t)\sin\varphi(t)$ with a low-pass filter of bandwidth $1/\tau$ Hz. The spectrum of $R(t)\sin\varphi(t)$ is easily given if the resulting noise voltage $R(t)\cos[2\pi f_o t-\varphi(t)]$ is analyzed into its individual frequency components, as indicated in Figure 1.

The frequency range f_1 to f_2 is subdivided into small intervals Δf. The frequency of one such subregion differs from the mean frequency f_o by a certain multiple of Δf. The contribution of each of these Δf intervals can be stated as a cosine wave of definite amplitude and phase. Therefore, if $B = f_2 - f_1$ is subdivided into $2n + 1$ intervals of width Δf:

$$R(t)\cos[2\pi f_o t-\varphi(t)] = \sum_{i=-n}^{n} c_i \cos[2\pi(f_o-i\Delta f)t-\varphi_i] \quad (3)$$

If equation (3) is multiplied by $2\sin 2\pi f_o t$, it is easily seen that there appears a spectrum of constant power density, beginning at frequency 0 and extending to the frequency $B/2 = f_2 - f_0 = f_0 - f_1$, along with a spectrum of width B with $2f_0$ as the middle frequency. That spectrum is not of interest here. Then we have

$$2V'(t)\sin 2\pi f_o t = R(t)\sin\varphi(t) + c_s + \text{terms of frequency } 2f_o$$

$$= \sum_{i=-n}^{n} c_i \sin[2\pi i\Delta f t + \varphi_i] + c_s + \text{terms of frequency } 2f_o \tag{4}$$

$$= \sum_{i=o}^{n} c_i \sin[2\pi i\Delta f t + \varphi_i] - \sum_{i=1}^{n} c_{-i} \sin[2\pi i\Delta f t - \varphi_{-i}] +$$

$$+ c_s + \text{terms of frequency } 2f_o$$

According to the formulas

$$\sin(\alpha \pm \beta) = \sin\alpha \cos\beta \pm \cos\alpha \sin\beta \quad \text{and}$$

$$a \sin\alpha + b \cos\alpha = \sqrt{a^2+b^2}\,\sin(\alpha + \text{arc tg}\frac{b}{a})$$

The two terms of the same frequency in equation (4) can be combined. Thus we derive the square-wave spectrum of $R(t)\sin\varphi(t)$ if we consider that the root-mean-square values of all c_i are equal.

In the conventional synchronous amplifier, therefore, the signal appearing as an alternating voltage is converted into a direct voltage by multiplication with a reference voltage. Then a low-pass, rather than a band-pass, filter is needed to limit the noise bandwidth. The bandwidth of a low-pass filter is given by the reciprocal time constant, so that very narrow bandwidths can be obtained quite easily. The principle has been applied in commercial devices. For example, they allow detection at 1 kHz bandwidth of a signal which is 40 dB below the noise level.

2. Modification of the Synchronous Amplifier

In the procedure for synchronous amplification which was briefly described in the preceding section, we worked with a signal of known frequency and constant phase, so that a reference voltage of identical frequency and phase could be produced, which by multiplication converts the signal voltage into a direct voltage. The frequency and phase of the signal voltage are determined, for example, by a chopper disk. The function of the chopper disk is to modulate the radiation which falls steadily upon it so as to obtain an alternating voltage, with no direct component, as the signal voltage. Now, one may ask why the signal voltage is first converted into an alternating voltage so that it can be reconverted to direct voltage in a synchronous amplifier. There are primarily two reasons: first, the amplification of small alternating voltages is significantly less expensive than amplification of direct voltages; second, the detectors usually employed are semiconductors which have very high inherent noise at low frequencies, i. e., near a direct voltage, decreasing generally as 1/f. For instance, at 1 kHz, common chopper frequency, the inherent noise is very significantly less that at zero frequency.

Because of the time constants of detectors, the chopper frequency cannot be made as high as might be desired. But in every case, the signal frequency of infrared radiometers is determined by the chopper. That is, it is produced in the receiver, and is particularly variable and controllable. This is possible, of course, only with some certain inertia when mechanical choppers (rotating wheels with teeth or apertures, vibrating springs or reeds) are used. Electronic choppers for infrared radiation can be controlled significantly more rapidly [3]. Another chopper technique is used in other possible applications of the process described below; e. g., in microwave radiometry. We will not concern ourselves with details here. Rather, we will presume for the procedure designated here as "modified synchronous amplification" that the signal phase can be controlled with definite small time constants, and that the information is contained in the signal amplitude, so that it is not affected by the phase control.

We consider the two voltages:

$$V(t) = R(t)\cos[2\pi f_o t-\varphi(t)] \quad \text{and} \tag{5}$$

$$V^+(t) = V(t) + c_s\sin[2\pi f_o t-\varphi_s(t)] \tag{6}$$

As in the previous section, Equation (5) gives the narrow-band noise voltage. The voltage $V^+(t)$ differs from $V(t)$ in that the signal voltage $c_s\sin[2\pi f_o t-\varphi_s(t)]$ is added to it. Let the phase $\varphi_s(t)$ of the signal remain undecided at first. Because signal plus noise always appears at the detector, the voltage $V(t)$ cannot be measured directly. For a small signal-to-noise ratio, as is assumed here, the difference $V^+(t) - V(t)$ is therefore a small value, so that V^+ is an approximation for $V(t)$. Now let φ_s be controlled so that, as exactly as possible,

$$\varphi(t) \approx \varphi_s(t) = \varphi'(t) \tag{7}$$

The control loop is described in more detail in the next section. The voltage measured at the amplifier output is then:

$$V^+(t) = R(t)\cos[2\pi f_o t-\varphi(t)] + c_s\sin[2\pi f_o t-\varphi'(t)] \tag{8}$$

Since f_0 is known, a reference voltage $2\sin[2\pi f_o t-\varphi'(t)]$ can be produced.

We multiply this with equation (8) and obtain:

$$2V^+(t)\sin[2\pi f_o t-\varphi'(t)] = R(t)\sin[\varphi(t)-\varphi'(t)] + c_s + \text{terms of frequency } 2f_o \tag{9}$$

The terms of the frequency $2f_0$ are again filtered out. Also, $\varphi(t)-\varphi'(t)$ is a relatively small angle, so that the sine can be replaced by the angle.

We have left

$$R(t)[\varphi(t)-\varphi'(t)] + c_s \qquad (10)$$

as the measured voltage.

The residual noise with the signal voltage c_s is therefore determined by the function $\varphi(t)-\varphi'(t)$, the difference between the actual noise phase and the controlled phase angle of the reference voltage. If the control is very accurate, then $\varphi-\varphi'$, and thus the noise voltage is very small. For the rest, of course, the noise component $R(t)[\varphi-\varphi']$ of equation (10) is not fully effective as a disturbing voltage. Rather, it is only the frequency components directly adjacent to zero frequency from the spectrum of $R(t)[\varphi-\varphi']$ which are filtered out by the indicator. Without more exact analysis of the measuring and controlling technology to be used, it is difficult to say more about $\varphi(t)-\varphi'(t)$. Still, it is not unreasonable to assume that the value of $\varphi-\varphi'$ will not exceed a few degrees (about 0.1 radian). As we know, $\varphi(t)$ varies statistically and irregularly with a time constant $\approx 1/B$ (B = bandwidth). Thus it is quite obvious to assume that the function $\varphi-\varphi'$ also fluctuates statistically with the time constant, or correlation time, 1/B.

For the conventional synchronous amplifier we had instead of Equation (10)

$$R(t)\sin\varphi(t) + c_s \qquad (2)$$

We derived in the previous section the spectrum of the noise voltage $R(t)\sin\varphi$ appearing here. With regard to the Equation (10) which is of primary interest to us, we now estimate the spectrum with reference to the correlation function. This is equally applicable to Equations (2) and (10), and allows a comparison:

$R(t)$ and $\varphi(t)$ vary with the time constant (= correlation time) 1/B. Thus the autocorrelation function $\phi(s)$ will have the value zero for displacement values, s, greater than 1/B. A Fourier transformation of the autocorrelation function $\phi(s)$ yields the power spectrum; i. e., the square of the amplitude. The values for $s = 0$ are:

$$\begin{aligned}
\phi_k(0) &= \overline{\left\{R(t)\sin\varphi(t)\right\}^2} && \text{for conventional synchronous amplification} \\
\phi_m(0) &= \overline{\left\{R(t)[\varphi-\varphi']\right\}^2} && \text{for modified synchronous amplification}
\end{aligned} \tag{11}$$

In both cases, the value of $\phi(0)$ for $s = 0$ drops to zero within the short time 1/B. The exact curve of the drop-off is not very important for the components of the low-frequency spectrum near zero frequency [4]. Thus the autocorrelation function is a quasi-representation of a brief pulse, as sketched in Figure 2. The spectrum is the Fourier transformation of this pulse, and the amplitudes of the Fourier transformation of a pulse are proportional to the area of the pulse. That is, here they are proportional to $\phi_k(0)/B$ or $\phi_m(0)/B$. The course of the curve can be, and is, different for conventional and synchronous amplification. The ratio of the "pulse areas" of the autocorrelation function is therefore not simply equal to the ratio $[\phi_k(0)/B] : [\phi_m(0)/B] = \phi_k(0) : \phi_m(0)$, but this value is a certain approximation. The improvement factor, i. e., the ratio of the perturbing noise amplitude for conventional and modified synchronous amplification, is approximately given, then, by

$$\sqrt{\frac{\phi_k(0)}{\phi_m(0)}} \approx \text{Improvement Factor} \tag{12}$$

This expression can still be simplified. From Equation (12), $\phi(0)$ is the mean value of the product of two statistically independent functions. According to [5] this is equal to the product of the means of the factors. Therefore:

$$\begin{aligned}
\overline{\left\{R(t)\sin\varphi(t)\right\}^2} &= \overline{[R(t)]^2}\,\overline{\sin^2\varphi(t)} = \overline{[R(t)]^2}\,\frac{1}{2} \\
\overline{\left\{R(t)[\varphi-\varphi']\right\}^2} &= \overline{[R(t)]^2}\,\overline{[\varphi-\varphi']^2}
\end{aligned} \tag{13}$$

and

$$\sqrt{\frac{\phi_k(0)}{\phi_m(0)}} = 1 \bigg/ \sqrt{2[\varphi-\varphi']^2} \approx \text{Improvement Factor} \qquad (14)$$

For $\varphi-\varphi'$ we had assumed a value smaller than 0.1, so that according to Equation (14), we would obtain an improvement factor greater than 10: $\sqrt{2}$ = 7. To repeat once more: this is a very crudely estimated value, without any analysis of measuring and control technology. The more accurate the measuring and control process, the greater the improvement in accuracy.

3. The Control.

In the preceding it was shown that an improvement of the signal-to-noise ratio is possible if a reference voltage can be adjusted as exactly as possible orthogonal to the noise voltage, i. e., displaced 90° in phase, and if the signal voltage can be adjusted parallel to this reference. We will not consider the control process as such more extensively.

Even if the resulting voltage at low signal-to-noise ratio, $V^+ = c_s \sin(2\pi f_o t - \varphi_s) + V$ (6) differs by only a small amount from the noise voltage V, the control cannot be based simply on the resulting voltage, or the signal voltage will be controlled out along with the noise voltage. A reference voltage V_{ref} which is orthogonal to the resultant V^+ (i. e., displaced in phase by 90°) would always yield zero on multiplication (components of the frequency $2f_0$ are neglected here and in the following).

The diagram of Figure 3 may clarify the conditions more. If it is possible to control the reference voltage V_{ref} orthogonal to the noise voltage V, as shown in Figure 3a, then the noise can be suppressed by multiplication, down to a residual noise determined by the accuracy of control. If the signal c_s is largely parallel to V_{ref}, the signal voltage is maintained through the multiplication. These are the operating conditions to be sought. If, on the other hand, as shown in Figure 3b, V_{ref} is orthogonal to the resultant V^+, then with absolutely exact control the value of zero is obtained, independent of the magnitude of the signal voltage c_s. The stochastically assumed control errors also cause a noise spectrum here. Its amplitude becomes smaller, the less the divergence. But the full amount of the signal is not obtained. Rather, it is controlled out more toward zero.

Therefore a criterion must be found for the control, which is based on the phase angle of the noise voltage V and not on the phase angle of the resultant V^+. There is a certain similarity or relationship with the "phase-locked loop" technique developed in the USA and often applied in recent years [6]. Therefore the phase control loop of the modified synchronous amplifier will be considered in imitation of the theory applied in that case. The goal is an estimate of the particularly interesting function $\varphi(t) - \varphi'(t)$, which we shall designate here as F(t). Thus we shall investigate whether it is possible to control the reference voltage orthogonal to the narrow band noise voltage if we do not have available as a control criterion the phase of the noise voltage, somehow measured, but the output voltage following the multiplier. In addition, we leave untouched a narrow frequency region around zero frequency, because this is to be reserved for the signal indication.

As previously, we designate the noise voltage as $V(t) = R(t)\cos[2\pi f_o t-\varphi(t)]$ and the reference voltage as $V_{ref} = 2\sin[2\pi f_o t-\varphi'(t)]$. Control is provided by the scheme of Figure 4.

The multiplier yields:

$$\begin{aligned} V(t)\,V_{ref}(t) &= R(t)\cos[2\pi f_o t-\varphi(t)]\;2\sin[2\pi f_o t-\varphi'(t)] \\ &= R(t)\sin[\varphi(t)-\varphi'(t)] + \text{terms of frequency } 2f_o \end{aligned} \tag{15}$$

The terms of frequency $2f_0$ are to be filtered out, and will be neglected here and in the following.

Let us again assume that the control has a certain accuracy, so that $\sin[\varphi(t)-\varphi'(t)]$ can be replaced by $[\varphi(t)-\varphi'(t)]$. The control signal input to the "voltage-controlled oscillator", $x(t)$, is then

$$x(t) = R(t)[\varphi(t)-\varphi'(t)] = R(t)F(t) \tag{16}$$

Equation (16) applies at first without the sparing of a narrow frequency region around zero frequency, which we have mentioned (and which is necessary for signal indication). Let the voltage-controlled oscillator have the resting frequency f_0, and let it react to an input control voltage with the phase change

$$\frac{d\varphi'(t)}{dt} = k\, x(t) \tag{17}$$

Then from (15), (16) and (17), we obtain the differential equation

$$\frac{dF(t)}{dt} = \frac{d}{dt}[\varphi(t)-\varphi'(t)] = \frac{d\varphi(t)}{dt} - k\, x(t) = \frac{d\varphi(t)}{dt} - k\, R(t)F(t) \tag{18}$$

or

$$F(t) + k\, R(t)F(t) = \frac{d\varphi(t)}{dt}$$

To solve this differential equation, we assume that the control error $F(0) = 0$ at time $t = 0$. Furthermore, let the control have a time constant small enough that $R(t)$ and $d\varphi(t)/dt$ may be considered constant. (The signal phase changes with a time constant $1/B$, because the signal must pass the amplifier with the bandwidth B. The signal phase thus follows the reference phase determined with the time constant $1/B$, according to the principle of the modified synchronous amplifier.) Under these conditions, the solution of (18) is

$$F(t) = [\varphi(t)-\varphi'(t)] = \frac{d\varphi(t)/dt}{k\, R(t)}\,[1 - e^{-kR(t)t}] \tag{19}$$

After a certain time the exponential term becomes zero. Then the control error can be set as

$$F(t) = [\varphi(t)-\varphi'(t)] \approx \frac{d\varphi(t)/dt}{k\ R(t)} \qquad (kR(t)t \gg 1) \tag{20}$$

As expected, it becomes smaller, the greater the control constant k is. Besides this, we note that on the average $R(t)$ is proportional to $\sqrt{B}$. The correlation time is given by $1/B$, so that $d\varphi(t)/dt$ may well be proportional to B. With k fixed, the control error given by (20) would be proportional to $\sqrt{B}$, but k can be assigned so that it can be canceled out.

Now we assume that the frequency region to be reserved near zero frequency for the signal indication, which has already been mentioned, is set aside. This means that all the lower frequencies are missing for the control signal $x(t)$ in (16). Then, instead of $x(t)$ we have:

$$x_{\bullet}(t) = x(t) - \gamma(t) = R(t)\ F(t) - \gamma(t) \tag{21}$$

in which $\gamma(t)$ represents a function of time which changes slowly, not only in comparison to the control time constant, but also in comparison to $1/B$. It changes so slowly that we can consider it as a constant on insertion into the differential equation. Instead of (18), then, we obtain:

$$\begin{aligned} \dot{F}(t) &= \frac{d\varphi(t)}{dt} - k\,x_{\bullet}(t) = \frac{d\varphi(t)}{dt} - k\,[R(t)F(t) - \gamma(t)] \\ \dot{F}(t) + k\,R(t)F(t) &= \frac{d\varphi(t)}{dt} + k\,\gamma(t) \end{aligned} \tag{22}$$

with the solution

$$F(t) = \frac{[d\varphi(t)/dt] + k\gamma(t)}{k\ R(t)} \circ [1 - e^{-kR(t)t}] \tag{23}$$

The lag error, which appears after a certain time in this case, after the exponential term has become zero, then becomes

$$F(t) = [\varphi(t)-\varphi'(t)] \quad \frac{d\varphi(t)/dt}{k\,R(t)} + \frac{\gamma(t)}{R(t)} \qquad (\,kR(t)t \gg 1\,) \tag{24}$$

From Equation (24) we learn that the additional control error $\gamma(t)/R(t)$ which appears because of the elimination of a narrow frequency range near zero frequency can be made small by appropriately large bandwidth, and correspondingly large $R(t)$. A large bandwidth, to be sure, causes a large $d\varphi(t)/dt$, but this can be compensated by a large control constant k.

Figure 5 shows a block diagram in which the process of modified synchronous amplification is used as an example with an infrared radiometer. It can be described as follows: The infrared radiation, which is assumed to be of constant intensity, falls on the optical modulator, which modulates the radiation with the frequency f_0 and the phase $\varphi_s(t)$. The modulated radiation arrives at the detector, in which the signal voltage is produced. A supplementary diaphragm allows the incident radiation to be restricted as desired. The amplified signal voltage was designated in the text as $c_s \sin[2\pi f_o t-\varphi_s(t)]$.
It is accompanied by a noise voltage $V(t) = R(t)\cos[2\pi f_o t-\varphi(t)]$. The total voltage $V^+(t) = V(t) + c_s \sin[2\pi f_o t-\varphi_s(t)]$ at the amplifier output is multiplied in a multiplier with the reference voltage $V_{ref}(t) = 2\sin[2\pi f_o t-\varphi_s(t)]$. The phase of this reference voltage is so controlled from a low-frequency filter through a control coupling that the low-frequency components at the multiplier output vanish, and so that the reference voltage $V_{ref}(t)$ and the noise voltage $V(t)$ are displaced by 90° in phase. A phase coupling ensures that the reference voltage $V_{ref}(t)$ and the signal voltage $c_s \sin[2\pi f_c t-\varphi_s(t)]$ always have the same phase [$(\varphi_s(t) = \varphi'(t)$, according to Equation (7)].

4. Concluding Remarks

The principle sketched here for a modification of the synchronous amplifier should not be considered a finished concept. Many parts of the problem, such as the phase distortion of the signal in the amplifier and the technical realization of the control loop, as well as the attainable control accuracy, have not yet been considered. Furthermore, bandwidth estimates and deliberations on the modulation frequency range are lacking. The only intent here was to pose for discussion an idea which in certain cases — perhaps — can improve noise suppression better than the conventional technique. It is clear from the beginning that a considerable expense would be required for practical realization of the process.

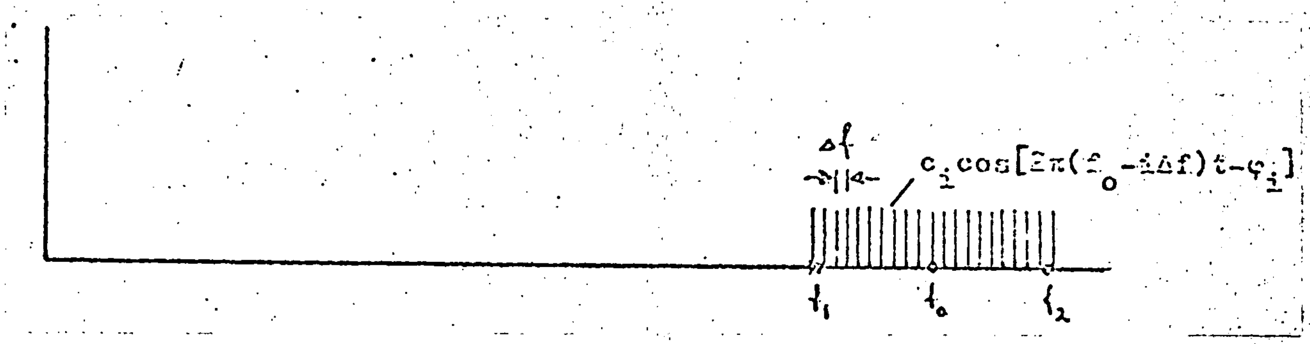

Figure 1. Subdivision of the filter pass band, with width $B = f_2 - f_1$, into small sub-bands Δf. Each of these sub-bands contributes to the resulting noise voltage.

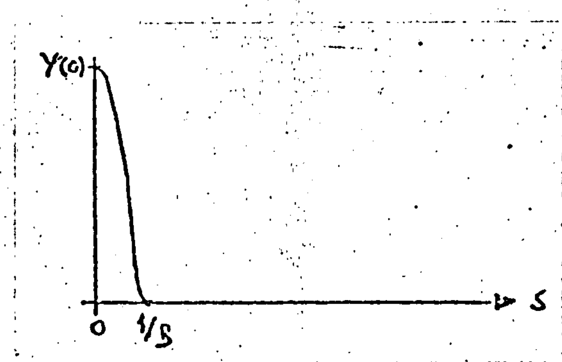

Figure 2. Schematic representation of the autocorrelation function of the noise voltage. This is valid not only for conventional, but also for modified synchronous amplification.

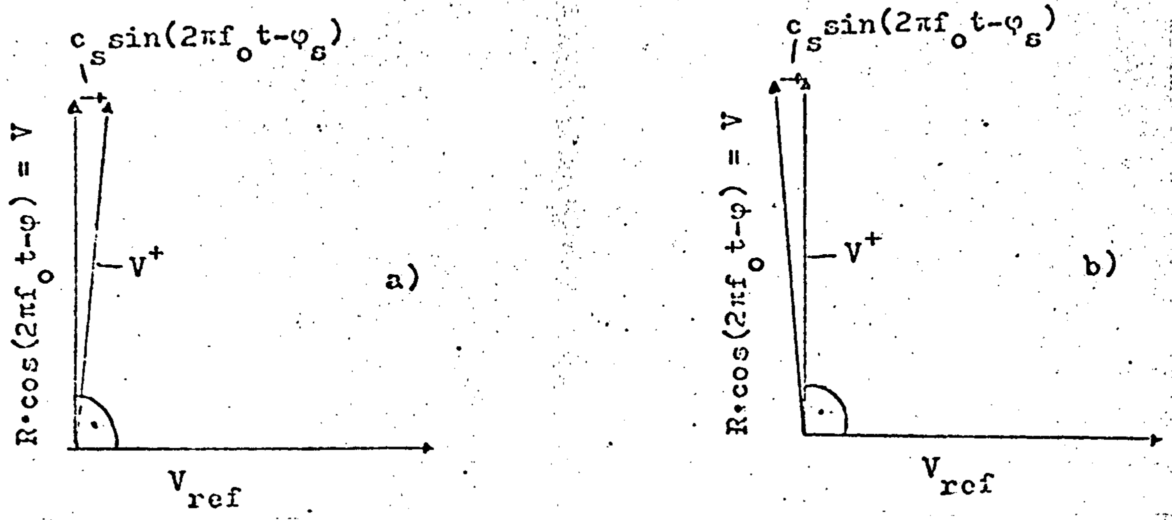

Figure 3. Vectorial representation of the voltages. In a) the reference voltage is orthogonal to the noise voltage. In b) the reference voltage is orthogonal to the sum of the noise and signal voltages.

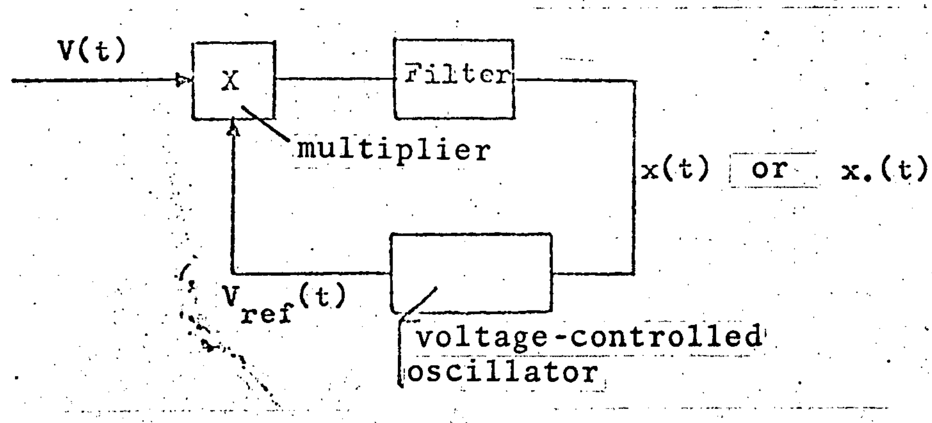

Figure 4. Block diagram of "phase-locked-loop" control [7].

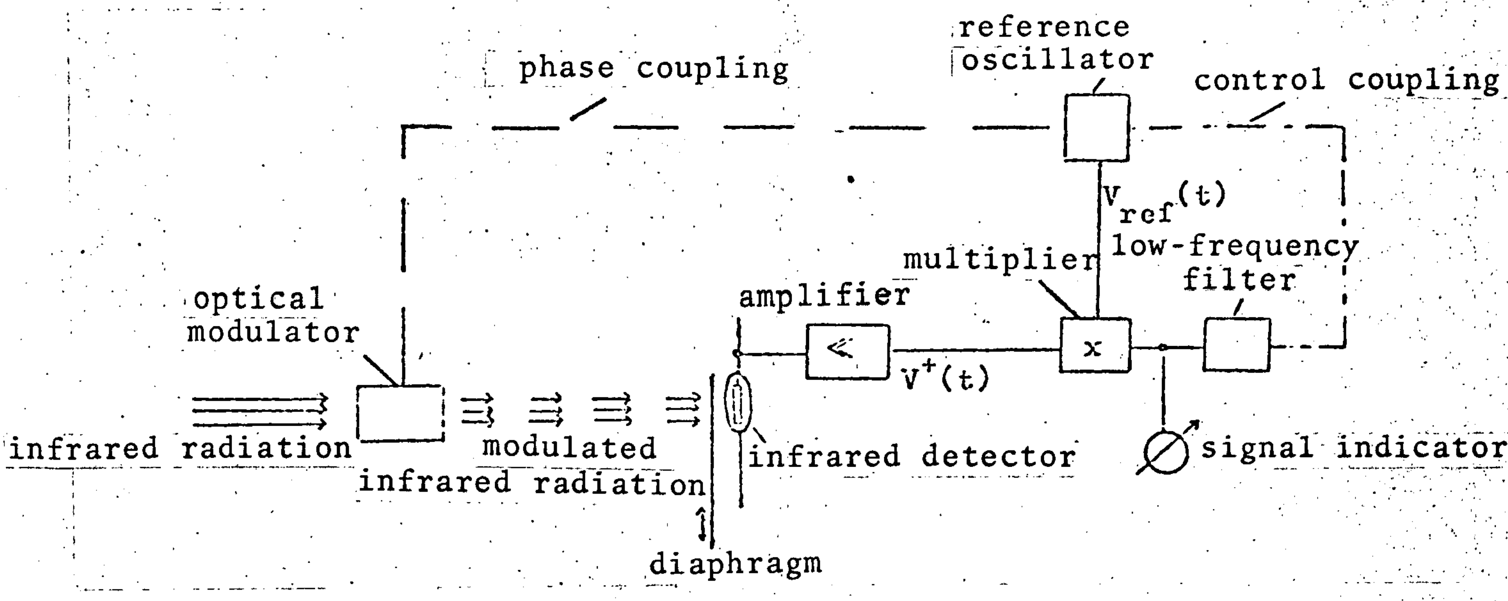

Figure 5. Block diagram of a process for modified synchronous amplification, explained using an infrared radiometer as an example.

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